

Package ‘CARBayes’

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Type Package

Title Spatial Generalised Linear Mixed Models for Areal Unit Data

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Description Implements a class of univariate and multivariate spatial generalised linear mixed models for areal unit data, with inference in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. The response variable can be binomial, Gaussian, multinomial, Poisson or zero-inflated Poisson (ZIP), and spatial autocorrelation is modelled by a set of random effects that are assigned a conditional autoregressive (CAR) prior distribution. A number of different models are available for univariate spatial data, including models with no random effects as well as random effects modelled by different types of CAR prior, including the BYM model (Besag et al. (1991) <doi:10.1007/BF00116466>), the Leroux model (Leroux et al. (2000) <doi:10.1007/978-1-4612-1284-3_4>) and the localised model (Lee et al. (2015) <doi:10.1002/env.2348>). Additionally, a multivariate CAR (MCAR) model for multivariate spatial data is available, as is a two-level hierarchical model for modelling data relating to individuals within areas. Full details are given in the vignette accompanying this package. The initial creation of this package was supported by the Economic and Social Research Council (ESRC) grant RES-000-22-4256, and ongoing development has been supported by the Engineering and Physical Science Research Council (EPSRC) grant EP/J017442/1, ESRC grant ES/K006460/1, Innovate UK / Natural Environment Research Council (NERC) grant NE/N007352/1 and the TB Alliance.

License GPL (>= 2)

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LazyLoad yes

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| | |
|----------------------------------|-----------|
| CARBayes-package | 2 |
| coef.CARBayes | 4 |
| combine.data.shapefile | 4 |
| fitted.CARBayes | 5 |
| highlight.borders | 6 |
| logLik.CARBayes | 7 |
| model.matrix.CARBayes | 7 |
| MVS.CARleroux | 8 |
| print.CARBayes | 11 |
| residuals.CARBayes | 12 |
| S.CARbym | 13 |
| S.CARdissimilarity | 16 |
| S.CARleroux | 19 |
| S.CARlocalised | 22 |
| S.CARmultilevel | 25 |
| S.glm | 28 |
| summarise.lincomb | 31 |
| summarise.samples | 32 |
| Index | 33 |

CARBayes-package

Spatial Generalised Linear Mixed Models for Areal Unit Data

Description

Implements a class of univariate and multivariate spatial generalised linear mixed models for areal unit data, with inference in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. The response variable can be binomial, Gaussian, multinomial, Poisson or zero-inflated Poisson (ZIP), and spatial autocorrelation is modelled by a set of random effects that are assigned a conditional autoregressive (CAR) prior distribution. A number of different models are available for univariate spatial data, including models with no random effects as well as random effects modelled by different types of CAR prior, including the BYM model (Besag et al. (1991) <doi:10.1007/BF00116466>), the Leroux model (Leroux et al. (2000) <doi:10.1007/978-1-4612-1284-3_4>) and the localised model (Lee et al. (2015) <doi:10.1002/env.2348>). Additionally, a

multivariate CAR (MCAR) model for multivariate spatial data is available, as is a two-level hierarchical model for modelling data relating to individuals within areas. Full details are given in the vignette accompanying this package. The initial creation of this package was supported by the Economic and Social Research Council (ESRC) grant RES-000-22-4256, and on-going development has been supported by the Engineering and Physical Science Research Council (EPSRC) grant EP/J017442/1, ESRC grant ES/K006460/1, Innovate UK / Natural Environment Research Council (NERC) grant NE/N007352/1 and the TB Alliance.

Details

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Author(s)

Maintainer: Duncan Lee <Duncan.Lee@glasgow.ac.uk>

References

- Besag, J. and York, J and Mollie, A (1991). Bayesian image restoration with two applications in spatial statistics. *Annals of the Institute of Statistics and Mathematics* 43, 1-59.
- Gelfand, A and Vounatsou, P (2003). Proper multivariate conditional autoregressive models for spatial data analysis, *Biostatistics*, 4, 11-25.
- Kavanagh, L., D. Lee, and G. Pryce (2016). Is Poverty Decentralising? Quantifying Uncertainty in the Decentralisation of Urban Poverty, *Annals of the American Association of Geographers*, 106, 1286-1298.
- Lee, D. and Mitchell, R (2012). Boundary detection in disease mapping studies. *Biostatistics*, 13, 415-426.
- Lee, D and Sarran, C (2015). Controlling for unmeasured confounding and spatial misalignment in long-term air pollution and health studies, *Environmetrics*, 26, 477-487.
- Leroux B, Lei X, Breslow N (2000). "Estimation of Disease Rates in SmallAreas: A New Mixed Model for Spatial Dependence." In M Halloran, D Berry (eds.), *Statistical Models in Epidemiology, the Environment and Clinical Trials*, pp. 179-191. Springer-Verlag, New York.
- Roberts, G and Rosenthal, J (1998). Optimal scaling of discrete approximations to the Langevin diffusions, *Journal of the Royal Statistical Society Series B* 60, 255-268.

Examples

```
## See the examples in the function specific help files and in the vignette  
## accompanying this package.
```

coef.CARBayes *Extract the regression coefficients from a model.*

Description

This function takes a CARBayes object and returns the vector of estimated regression coefficients (posterior medians).

Usage

```
## S3 method for class 'CARBayes'  
coef(object, ...)
```

Arguments

object A CARBayes fitted model object.
... Ignored.

Author(s)

Duncan Lee

combine.data.shapefile
*Combines a data frame with a shapefile to create a SpatialPolygons-
DataFrame object.*

Description

This function combines a data frame with a shapefile to create a SpatialPolygonsDataFrame object from the 'sp' package. The creation of this object allows the variables in the data frame to be mapped using the 'splot()' function, and the neighbourhood matrix W to be created using the 'poly2nb' and 'nb2mat' functions. An example is given in the vignette accompanying this package. The mapping of the data to the shapefile is done by matching the rownames of the data frame to the first column in the dbf file.

Usage

```
combine.data.shapefile(data, shp, dbf)
```

Arguments

| | |
|------|---|
| data | A data frame containing the variables relating to the K areas you wish to map or model. The row names of this data frame must appear in the first column of the dbf file. |
| shp | The .shp part of a shapefile containing the polygons for each of the K areas that the data relate to. |
| dbf | The .dbf part of the shapefile containing a lookup table whose first column should include the K row names of the data frame. |

Value

A SpatialPolygonsDataFrame object from the sp package containing the combined data and shapefile object.

Author(s)

Duncan Lee

Examples

```
## See the vignette accompanying this package for an example of its use.
```

```
fitted.CARBayes      Extract the fitted values from a model.
```

Description

This function takes a CARBayes object and returns the vector of fitted values (posterior means).

Usage

```
## S3 method for class 'CARBayes'  
fitted(object, ...)
```

Arguments

| | |
|--------|---------------------------------|
| object | A CARBayes fitted model object. |
| ... | Ignored. |

Author(s)

Duncan Lee

| | |
|-------------------|---|
| highlight.borders | <i>Creates a SpatialPoints object identifying a subset of borders between neighbouring areas.</i> |
|-------------------|---|

Description

Creates a SpatialPoints object identifying a subset of borders between neighbouring areas, which allows them to be overlaid on a map. An example is given in the vignette accompanying this package.

Usage

```
highlight.borders(border.locations, spdata)
```

Arguments

| | |
|------------------|---|
| border.locations | A K by K matrix, where K is the number of areas, containing 3 distinct values: NA for non-neighbouring areas; 0 for borders between neighbouring areas to be highlighted on a map; and 1 for borders between neighbouring areas not to be highlighted on a map. |
| spdata | The SpatialPolygonsDataFrame object used for plotting the data and creating the original neighbourhood matrix W. |

Value

A SpatialPoints object from the sp package, which contains the vertices of all the borders to be highlighted on the map. The mapping can be done using the leaflet package, see the vignette accompanying this package for an example.

Author(s)

Duncan Lee

Examples

```
## See the vignette accompanying this package for an example of its use.
```

| | |
|-----------------|---|
| logLik.CARBayes | <i>Extract the estimated loglikelihood from a fitted model.</i> |
|-----------------|---|

Description

This function takes a CARBayes object and returns the estimated loglikelihood (posterior means).

Usage

```
## S3 method for class 'CARBayes'  
logLik(object, ...)
```

Arguments

| | |
|--------|---------------------------------|
| object | A CARBayes fitted model object. |
| ... | Ignored. |

Author(s)

Duncan Lee

| | |
|-----------------------|--|
| model.matrix.CARBayes | <i>Extract the model (design) matrix from a model.</i> |
|-----------------------|--|

Description

This function takes a CARBayes object and returns the design matrix.

Usage

```
## S3 method for class 'CARBayes'  
model.matrix(object, ...)
```

Arguments

| | |
|--------|---------------------------------|
| object | A CARBayes fitted model object. |
| ... | Ignored. |

Author(s)

Duncan Lee

MVS.CARleroux

Fit a multivariate spatial generalised linear mixed model to data, where the random effects are modelled by a multivariate conditional autoregressive model.

Description

Fit a multivariate spatial generalised linear mixed model to areal unit data, where the response variable can be binomial, Gaussian, multinomial or Poisson. The linear predictor is modelled by known covariates and a vector of random effects. The latter account for both spatial and between variable correlation, via a Kronecker product formulation. Spatial correlation is captured by the conditional autoregressive (CAR) prior proposed by Leroux et al. (2000), and between variable correlation is captured by a between variable covariance matrix with no fixed structure. This is a type of multivariate conditional autoregressive (MCAR) model. Further details are given in the vignette accompanying this package. Independent (over space) random effects can be obtained by setting $\rho=0$, while the intrinsic MCAR model can be obtained by setting $\rho=1$. Inference is conducted in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. Missing (NA) values are allowed in the response, and posterior predictive distributions are created for the missing values using data augmentation. These are saved in the "samples" argument in the output of the function and are denoted by "Y". For the multinomial model the first category in the multinomial data (first column of the response matrix) is taken as the baseline, and the covariates are linearly related to the log of the ratio (θ_j / θ_1) for $j=1, \dots, J$, where θ_j is the probability of being in category j . For a full model specification see the vignette accompanying this package.

Usage

```
MVS.CARleroux(formula, family, data=NULL, trials=NULL, W, burnin,
n.sample, thin=1, prior.mean.beta=NULL, prior.var.beta=NULL, prior.nu2=NULL,
prior.Sigma.df=NULL, prior.Sigma.scale=NULL, rho=NULL, MALA=FALSE,
verbose=TRUE)
```

Arguments

| | |
|---------|---|
| formula | A formula for the covariate part of the model using the syntax of the <code>lm()</code> function. Offsets can be included here using the <code>offset()</code> function. The response and the offset (if included) should be matrices of dimension $K \times J$, where K is the number of spatial units and J is the number of different variables (categories in the multinomial model). The covariates should each be a $K \times 1$ vector, and different regression parameters are estimated for each of the J variables. Missing (NA) values are allowed in the response. |
| family | One of either "binomial", "gaussian", "multinomial", or "poisson", which respectively specify a binomial likelihood model with a logistic link function, a Gaussian likelihood model with an identity link function, a multinomial likelihood model with a logistic link function, or a Poisson likelihood model with a log link function. |
| data | An optional data.frame containing the variables in the formula. |

| | |
|--------------------------------|---|
| <code>trials</code> | Only used if <code>family="binomial"</code> or <code>family="multinomial"</code> . For the binomial family it is a $K \times J$ matrix the same dimension as the response. For the multinomial family it is a vector of length K . |
| <code>W</code> | A non-negative K by K neighbourhood matrix (where K is the number of spatial units). Typically a binary specification is used, where the jk th element equals one if areas (j, k) are spatially close (e.g. share a common border) and is zero otherwise. The matrix can be non-binary, but each row must contain at least one non-zero entry. |
| <code>burnin</code> | The number of MCMC samples to discard as the burn-in period. |
| <code>n.sample</code> | The number of MCMC samples to generate. |
| <code>thin</code> | The level of thinning to apply to the MCMC samples to reduce their temporal autocorrelation. Defaults to 1 (no thinning). |
| <code>prior.mean.beta</code> | A vector of prior means for the regression parameters <code>beta</code> (Gaussian priors are assumed). Defaults to a vector of zeros. |
| <code>prior.var.beta</code> | A vector of prior variances for the regression parameters <code>beta</code> (Gaussian priors are assumed). Defaults to a vector with values 100000. |
| <code>prior.nu2</code> | The prior shape and scale in the form of $c(\text{shape}, \text{scale})$ for an Inverse-Gamma(shape, scale) prior for <code>nu2</code> . Defaults to $c(1, 0.01)$ and only used if <code>family="Gaussian"</code> . |
| <code>prior.Sigma.df</code> | The prior degrees of freedom for the Inverse-Wishart prior for <code>Sigma</code> . Defaults to $J+1$. |
| <code>prior.Sigma.scale</code> | The prior J times J scale matrix for the Inverse-Wishart prior for <code>Sigma</code> . Defaults to the identity matrix divided by 1000. |
| <code>rho</code> | The value in the interval $[0, 1]$ that the spatial dependence parameter <code>rho</code> is fixed at if it should not be estimated. If this argument is <code>NULL</code> then <code>rho</code> is estimated in the model. |
| <code>MALA</code> | Logical, should the function use Metropolis adjusted Langevin algorithm (MALA) updates (<code>TRUE</code>) or simple random walk (<code>FALSE</code> , default) updates for the regression parameters and the random effects. If <code>family="gaussian"</code> the MALA argument only applies to the random effects as the regression parameters are Gibbs sampled. Not applicable if <code>family="multinomial"</code> where random walk updates are used. |
| <code>verbose</code> | Logical, should the function update the user on its progress. |

Value

| | |
|------------------------------|---|
| <code>summary.results</code> | A summary table of the parameters. |
| <code>samples</code> | A list containing the MCMC samples from the model. |
| <code>fitted.values</code> | A matrix of fitted values for each area and response variable. |
| <code>residuals</code> | A list with 2 elements, where each element is a matrix of a type of residuals. Each row of a matrix relates to an area and each column to a response (category). The types of residual are "response" (raw), and "pearson". |

| | |
|---------------------|--|
| modelfit | Model fit criteria including the Deviance Information Criterion (DIC) and its corresponding estimated effective number of parameters (p.d), the Log Marginal Predictive Likelihood (LMPL), the Watanabe-Akaike Information Criterion (WAIC) and its corresponding estimated number of effective parameters (p.w), and the loglikelihood. |
| accept | The acceptance probabilities for the parameters. |
| localised.structure | NULL, for compatability with other models. |
| formula | The formula (as a text string) for the response, covariate and offset parts of the model |
| model | A text string describing the model fit. |
| X | The design matrix of covariates. |

Author(s)

Duncan Lee

References

Gelfand, A and Vounatsou, P (2003). Proper multivariate conditional autoregressive models for spatial data analysis, *Biostatistics*, 4, 11-25.

Kavanagh, L., D. Lee, and G. Pryce (2016). Is Poverty Decentralising? Quantifying Uncertainty in the Decentralisation of Urban Poverty, *Annals of the American Association of Geographers*, 106, 1286-1298.

Leroux B, Lei X, Breslow N (2000). "Estimation of Disease Rates in SmallAreas: A New Mixed Model for Spatial Dependence." In M Halloran, D Berry (eds.), *Statistical Models in Epidemiology, the Environment and Clinical Trials*, pp. 179-191. Springer-Verlag, New York.

Examples

```
#####
#### Run the model on simulated data on a lattice
#####
#### Load other libraries required
library(MASS)

#### Set up a square lattice region
x.easting <- 1:10
x.northing <- 1:10
Grid <- expand.grid(x.easting, x.northing)
K <- nrow(Grid)

#### set up distance and neighbourhood (W, based on sharing a common border) matrices
distance <- as.matrix(dist(Grid))
W <-array(0, c(K,K))
W[distance==1] <-1
K <- nrow(W)

#### Generate the correlation structures
```

```

Q.W <- 0.99 * (diag(apply(W, 2, sum)) - W) + 0.01 * diag(rep(1,K))
Q.W.inv <- solve(Q.W)

Sigma <- matrix(c(1,0.5,0, 0.5,1,0.3, 0, 0.3, 1), nrow=3)
Sigma.inv <- solve(Sigma)
J <- nrow(Sigma)
N.all <- K * J

precision.phi <- kronecker(Q.W, Sigma.inv)
var.phi <- solve(precision.phi)

#### Generate the covariate component
x1 <- rnorm(K)
x2 <- rnorm(K)
XB <- cbind(0.1 * x1 - 0.1*x2, -0.1 * x1 + 0.1*x2, 0.1 * x1 - 0.1*x2)

#### Generate the random effects
phi <- mvrnorm(n=1, mu=rep(0,N.all), Sigma=var.phi)

#### Generate the response data
lp <- as.numeric(t(XB)) + phi
prob <- exp(lp) / (1 + exp(lp))
trials.vec <- rep(100,N.all)
Y.vec <- rbinom(n=N.all, size=trials.vec, prob=prob)

#### Turn the data and trials into matrices where each row is an area.
Y <- matrix(Y.vec, nrow=K, ncol=J, byrow=TRUE)
trials <- matrix(trials.vec, nrow=K, ncol=J, byrow=TRUE)

#### Run the Leroux model
formula <- Y ~ x1 + x2
## Not run: model <- MVS.CARleroux(formula=formula, family="binomial",
trials=trials, W=W, burnin=20000, n.sample=100000)
## End(Not run)

#### Toy example for checking
model <- MVS.CARleroux(formula=formula, family="binomial",
trials=trials, W=W, burnin=10, n.sample=50)

```

print.CARBayes

Print a summary of a fitted CARBayes model to the screen.

Description

This function takes a CARBayes object and returns a summary of the fitted model. The summary includes, for selected parameters, posterior medians and 95 percent credible intervals, the effective number of independent samples and the Geweke convergence diagnostic in the form of a Z-score.

Usage

```
## S3 method for class 'CARBayes'  
print(x, ...)
```

Arguments

| | |
|-----|---------------------------------|
| x | A CARBayes fitted model object. |
| ... | Ignored. |

Author(s)

Duncan Lee

residuals.CARBayes *Extract the residuals from a model.*

Description

This function takes a CARBayes object and returns a set of residuals. The allowable types of residual are "response" (raw) and "pearson" (the default). In each case the fitted values are based on posterior means.

Usage

```
## S3 method for class 'CARBayes'  
residuals(object, type, ...)
```

Arguments

| | |
|--------|---|
| object | A CARBayes fitted model object. |
| type | A text string and one of "response" or "pearson". If this argument is omitted the default is "pearson". |
| ... | Ignored. |

Author(s)

Duncan Lee

S. CARbym

Fit a spatial generalised linear mixed model to data, where the random effects have a BYM conditional autoregressive prior.

Description

Fit a spatial generalised linear mixed model to areal unit data, where the response variable can be binomial, Poisson, or zero-inflated Poisson (ZIP). Note, a Gaussian likelihood is not allowed because of a lack of identifiability among the parameters. The linear predictor is modelled by known covariates and 2 vectors of random effects. The latter are modelled by the BYM conditional autoregressive prior proposed by Besag et al. (1991), and further details are given in the vignette accompanying this package. Inference is conducted in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. Missing (NA) values are allowed in the response, and posterior predictive distributions are created for the missing values using data augmentation. These are saved in the "samples" argument in the output of the function and are denoted by "Y". For the ZIP model covariates can be used to estimate the probability of an observation being a structural zero, via a logistic regression equation. For a full model specification see the vignette accompanying this package.

Usage

```
S.CARbym(formula, formula.omega=NULL, family, data=NULL, trials=NULL, W, burnin,
n.sample, thin=1, prior.mean.beta=NULL, prior.var.beta=NULL, prior.tau2=NULL,
prior.sigma2=NULL, prior.mean.delta=NULL, prior.var.delta=NULL, MALA=FALSE,
verbose=TRUE)
```

Arguments

| | |
|---------------|--|
| formula | A formula for the covariate part of the model using the syntax of the <code>lm()</code> function. Offsets can be included here using the <code>offset()</code> function. The response, offset and each covariate are vectors of length $K*1$. The response can contain missing (NA) values. |
| formula.omega | A one-sided formula object with no response variable (left side of the "~") needed, specifying the covariates in the logistic regression model for modelling the probability of an observation being a structural zero. Each covariate (or an offset) needs to be a vector of length $K*1$. Only required for zero-inflated Poisson models. |
| family | One of either "binomial", "poisson" or "zip", which respectively specify a binomial likelihood model with a logistic link function, a Poisson likelihood model with a log link function, or a zero-inflated Poisson model with a log link function. |
| data | An optional data.frame containing the variables in the formula. |
| trials | A vector the same length as the response containing the total number of trials for each area. Only used if family="binomial". |

| | |
|-------------------------------|---|
| <code>W</code> | A non-negative K by K neighbourhood matrix (where K is the number of spatial units). Typically a binary specification is used, where the jkth element equals one if areas (j, k) are spatially close (e.g. share a common border) and is zero otherwise. The matrix can be non-binary, but each row must contain at least one non-zero entry. |
| <code>burnin</code> | The number of MCMC samples to discard as the burn-in period. |
| <code>n.sample</code> | The number of MCMC samples to generate. |
| <code>thin</code> | The level of thinning to apply to the MCMC samples to reduce their temporal autocorrelation. Defaults to 1 (no thinning). |
| <code>prior.mean.beta</code> | A vector of prior means for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector of zeros. |
| <code>prior.var.beta</code> | A vector of prior variances for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector with values 100000. |
| <code>prior.tau2</code> | The prior shape and scale in the form of c(shape, scale) for an Inverse-Gamma(shape, scale) prior for tau2. Defaults to c(1, 0.01). |
| <code>prior.sigma2</code> | The prior shape and scale in the form of c(shape, scale) for an Inverse-Gamma(shape, scale) prior for sigma2. Defaults to c(1, 0.01). |
| <code>prior.mean.delta</code> | A vector of prior means for the regression parameters delta (Gaussian priors are assumed) for the zero probability logistic regression component of the model. Defaults to a vector of zeros. |
| <code>prior.var.delta</code> | A vector of prior variances for the regression parameters delta (Gaussian priors are assumed) for the zero probability logistic regression component of the model. Defaults to a vector with values 100000. |
| <code>MALA</code> | Logical, should the function use Metropolis adjusted Langevin algorithm (MALA) updates (TRUE) or simple random walk (FALSE, default) updates for the regression parameters and random effects. |
| <code>verbose</code> | Logical, should the function update the user on its progress. |

Value

| | |
|------------------------------|--|
| <code>summary.results</code> | A summary table of the parameters. |
| <code>samples</code> | A list containing the MCMC samples from the model. |
| <code>fitted.values</code> | A vector of fitted values for each area. |
| <code>residuals</code> | A matrix with 2 columns where each column is a type of residual and each row relates to an area. The types are "response" (raw), and "pearson". |
| <code>modelfit</code> | Model fit criteria including the Deviance Information Criterion (DIC) and its corresponding estimated effective number of parameters (p.d), the Log Marginal Predictive Likelihood (LMPL), the Watanabe-Akaike Information Criterion (WAIC) and its corresponding estimated number of effective parameters (p.w), and the loglikelihood. |
| <code>accept</code> | The acceptance probabilities for the parameters. |

| | |
|---------------------|--|
| localised.structure | NULL, for compatability with other models. |
| formula | The formula (as a text string) for the response, covariate and offset parts of the model |
| model | A text string describing the model fit. |
| X | The design matrix of covariates. |

Author(s)

Duncan Lee

References

Besag, J., J. York, and A. Mollie (1991). Bayesian image restoration with two applications in spatial statistics. *Annals of the Institute of Statistics and Mathematics* 43, 1-59.

Examples

```
#####
#### Run the model on simulated data on a lattice
#####
#### Load other libraries required
library(MASS)

#### Set up a square lattice region
x.easting <- 1:10
x.northing <- 1:10
Grid <- expand.grid(x.easting, x.northing)
K <- nrow(Grid)

#### set up distance and neighbourhood (W, based on sharing a common border) matrices
distance <- as.matrix(dist(Grid))
W <- array(0, c(K,K))
W[distance==1] <-1

#### Generate the covariates and response data
x1 <- rnorm(K)
x2 <- rnorm(K)
theta <- rnorm(K, sd=0.05)
phi <- mvrnorm(n=1, mu=rep(0,K), Sigma=0.4 * exp(-0.1 * distance))
logit <- x1 + x2 + theta + phi
prob <- exp(logit) / (1 + exp(logit))
trials <- rep(50,K)
Y <- rbinom(n=K, size=trials, prob=prob)

#### Run the BYM model
formula <- Y ~ x1 + x2
## Not run: model <- S.CARbym(formula=formula, family="binomial", trials=trials,
W=W, burnin=20000, n.sample=100000)
## End(Not run)
```

```
#### Toy example for checking
model <- S.CARbym(formula=formula, family="binomial", trials=trials,
W=W, burnin=20, n.sample=50)
```

S.CARdissimilarity *Fit a spatial generalised linear mixed model to data, where the random effects have a localised conditional autoregressive prior.*

Description

Fit a spatial generalised linear mixed model to areal unit data, where the response variable can be binomial, Gaussian or Poisson. The linear predictor is modelled by known covariates and a vector of random effects. The latter are modelled by the localised conditional autoregressive prior proposed by Lee and Mitchell (2012), and further details are given in the vignette accompanying this package. Inference is conducted in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. Missing (NA) values are allowed in the response, and posterior predictive distributions are created for the missing values using data augmentation. These are saved in the "samples" argument in the output of the function and are denoted by "Y". For a full model specification see the vignette accompanying this package.

Usage

```
S.CARdissimilarity(formula, family, data=NULL, trials=NULL, W,
Z, W.binary=TRUE, burnin, n.sample, thin=1, prior.mean.beta=NULL,
prior.var.beta=NULL, prior.nu2=NULL, prior.tau2=NULL, MALA=FALSE, verbose=TRUE)
```

Arguments

| | |
|---------|---|
| formula | A formula for the covariate part of the model using the syntax of the <code>lm()</code> function. Offsets can be included here using the <code>offset()</code> function. The response, offset and each covariate is a vector of length $K \times 1$. The response can contain missing (NA) values. |
| family | One of either "binomial", "gaussian" or "poisson", which respectively specify a binomial likelihood model with a logistic link function, a Gaussian likelihood model with an identity link function, or a Poisson likelihood model with a log link function. |
| data | An optional data.frame containing the variables in the formula. |
| trials | A vector the same length as the response containing the total number of trials for each area. Only used if <code>family="binomial"</code> . |
| W | A non-negative K by K neighbourhood matrix (where K is the number of spatial units). Typically a binary specification is used, where the j th element equals one if areas (j, k) are spatially close (e.g. share a common border) and is zero otherwise. For this model only the matrix must be binary. |
| Z | A list, where each element is a K by K matrix of non-negative dissimilarity metrics. |

| | |
|------------------------------|---|
| <code>W.binary</code> | Logical, should the estimated neighbourhood matrix have only binary (0,1) values. |
| <code>burnin</code> | The number of MCMC samples to discard as the burn-in period. |
| <code>n.sample</code> | The number of MCMC samples to generate. |
| <code>thin</code> | The level of thinning to apply to the MCMC samples to reduce their temporal autocorrelation. Defaults to 1 (no thinning). |
| <code>prior.mean.beta</code> | A vector of prior means for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector of zeros. |
| <code>prior.var.beta</code> | A vector of prior variances for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector with values 100000. |
| <code>prior.nu2</code> | The prior shape and scale in the form of $c(\text{shape}, \text{scale})$ for an Inverse-Gamma(shape, scale) prior for nu2. Defaults to $c(1, 0.01)$ and only used if family="Gaussian". |
| <code>prior.tau2</code> | The prior shape and scale in the form of $c(\text{shape}, \text{scale})$ for an Inverse-Gamma(shape, scale) prior for tau2. Defaults to $c(1, 0.01)$. |
| <code>MALA</code> | Logical, should the function use Metropolis adjusted Langevin algorithm (MALA) updates (TRUE) or simple random walk (FALSE, default) updates for the regression parameters and random effects. Not applicable if family="gaussian". |
| <code>verbose</code> | Logical, should the function update the user on its progress. |

Value

| | |
|----------------------------------|---|
| <code>summary.results</code> | A summary table of the parameters. |
| <code>samples</code> | A list containing the MCMC samples from the model. |
| <code>fitted.values</code> | A vector of fitted values for each area. |
| <code>residuals</code> | A matrix with 2 columns where each column is a type of residual and each row relates to an area. The types are "response" (raw), and "pearson". |
| <code>modelfit</code> | Model fit criteria including the Deviance Information Criterion (DIC) and its corresponding estimated effective number of parameters (p.d), the Log Marginal Predictive Likelihood (LMPL), the Watanabe-Akaike Information Criterion (WAIC) and its corresponding estimated number of effective parameters (p.w), and the loglikelihood. |
| <code>accept</code> | The acceptance probabilities for the parameters. |
| <code>localised.structure</code> | A list containing two matrices: <code>W.posterior</code> contains posterior medians for each element w_{kj} of the K by K neighbourhood matrix W ; <code>W.border.prob</code> contains posterior probabilities that each w_{kj} element of the K by K neighbourhood matrix W equals zero. This corresponds to the posterior probability of a boundary in the random effects surface. The latter is only present if <code>W.binary=TRUE</code> , otherwise it is missing (NA). In all cases W elements that correspond to two non-neighbouring areas have NA values. |
| <code>formula</code> | The formula (as a text string) for the response, covariate and offset parts of the model |
| <code>model</code> | A text string describing the model fit. |
| <code>X</code> | The design matrix of covariates. |

Author(s)

Duncan Lee

References

Lee, D. and R. Mitchell (2012). Boundary detection in disease mapping studies. *Biostatistics*, 13, 415-426.

Examples

```
#####
#### Run the model on simulated data on a lattice
#####
#### Load other libraries required
library(MASS)

#### Set up a square lattice region
x.easting <- 1:10
x.northing <- 1:10
Grid <- expand.grid(x.easting, x.northing)
K <- nrow(Grid)

#### Split the area into two groups between which there will be a boundary.
groups <- rep(1, K)
groups[Grid$Var1>5] <- 2

#### set up distance and neighbourhood (W, based on sharing a common border) matrices
distance <- as.matrix(dist(Grid))
W <- array(0, c(K,K))
W[distance==1] <-1

#### Generate the response data
phi <- mvrnorm(n=1, mu=groups, Sigma=0.2 * exp(-0.1 * distance))
logit <- phi
prob <- exp(logit) / (1 + exp(logit))
trials <- rep(50,K)
Y <- rbinom(n=K, size=trials, prob=prob)

#### Generate a dissimilarity metric
dissimilarity <- cbind(groups) + rnorm(K, sd=0.2)
dissimilarity.matrix <- as.matrix(dist(cbind(dissimilarity, dissimilarity),
method="manhattan", diag=TRUE, upper=TRUE)) * W/2

Z <- list(dissimilarity.matrix=dissimilarity.matrix)

#### Run the localised smoothing model
formula <- Y ~ 1
## Not run: model <- S.CARdissimilarity(formula=formula, family="binomial",
trials=trials, W=W, Z=Z, W.binary=TRUE, burnin=20000, n.sample=100000)
## End(Not run)
```

```
#### Toy example for checking
model <- S.CARdisimilarity(formula=formula, family="binomial",
  trials=trials, W=W, Z=Z, W.binary=TRUE, burnin=10, n.sample=50)
```

S.CARleroux

Fit a spatial generalised linear mixed model to data, where the random effects have a Leroux conditional autoregressive prior.

Description

Fit a spatial generalised linear mixed model to areal unit data, where the response variable can be binomial, Gaussian, Poisson or zero-inflated Poisson (ZIP). The linear predictor is modelled by known covariates and a vector of random effects. The latter are modelled by the conditional autoregressive prior proposed by Leroux et al. (2000), and further details are given in the vignette accompanying this package. Independent random effects can be obtained by setting $\rho=0$, while the intrinsic CAR model can be obtained by setting $\rho=1$. Inference is conducted in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. Missing (NA) values are allowed in the response, and posterior predictive distributions are created for the missing values using data augmentation. These are saved in the "samples" argument in the output of the function and are denoted by "Y". For the ZIP model covariates can be used to estimate the probability of an observation being a structural zero, via a logistic regression equation. For a full model specification see the vignette accompanying this package.

Usage

```
S.CARleroux(formula, formula.omega=NULL, family, data=NULL, trials=NULL, W, burnin,
  n.sample, thin=1, prior.mean.beta=NULL, prior.var.beta=NULL,
  prior.nu2=NULL, prior.tau2=NULL, prior.mean.delta=NULL, prior.var.delta=NULL,
  rho=NULL, MALA=FALSE, verbose=TRUE)
```

Arguments

| | |
|---------------|--|
| formula | A formula for the covariate part of the model using the syntax of the <code>lm()</code> function. Offsets can be included here using the <code>offset()</code> function. The response, offset and each covariate is a vector of length $K*1$. The response can contain missing (NA) values. |
| formula.omega | A one-sided formula object with no response variable (left side of the "~") needed, specifying the covariates in the logistic regression model for modelling the probability of an observation being a structural zero. Each covariate (or an offset) needs to be a vector of length $K*1$. Only required for zero-inflated Poisson models. |
| family | One of either "binomial", "gaussian", "poisson" or "zip", which respectively specify a binomial likelihood model with a logistic link function, a Gaussian likelihood model with an identity link function, a Poisson likelihood model with a log link function, or a zero-inflated Poisson model with a log link function. |
| data | An optional data.frame containing the variables in the formula. |

| | |
|-------------------------------|---|
| <code>trials</code> | A vector the same length as the response containing the total number of trials for each area. Only used if <code>family="binomial"</code> . |
| <code>W</code> | A non-negative K by K neighbourhood matrix (where K is the number of spatial units). Typically a binary specification is used, where the jkth element equals one if areas (j, k) are spatially close (e.g. share a common border) and is zero otherwise. The matrix can be non-binary, but each row must contain at least one non-zero entry. |
| <code>burnin</code> | The number of MCMC samples to discard as the burn-in period. |
| <code>n.sample</code> | The number of MCMC samples to generate. |
| <code>thin</code> | The level of thinning to apply to the MCMC samples to reduce their temporal autocorrelation. Defaults to 1 (no thinning). |
| <code>prior.mean.beta</code> | A vector of prior means for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector of zeros. |
| <code>prior.var.beta</code> | A vector of prior variances for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector with values 100000. |
| <code>prior.nu2</code> | The prior shape and scale in the form of <code>c(shape, scale)</code> for an Inverse-Gamma(shape, scale) prior for nu2. Defaults to <code>c(1, 0.01)</code> and only used if <code>family="Gaussian"</code> . |
| <code>prior.tau2</code> | The prior shape and scale in the form of <code>c(shape, scale)</code> for an Inverse-Gamma(shape, scale) prior for tau2. Defaults to <code>c(1, 0.01)</code> . |
| <code>prior.mean.delta</code> | A vector of prior means for the regression parameters delta (Gaussian priors are assumed) for the zero probability logistic regression component of the model. Defaults to a vector of zeros. |
| <code>prior.var.delta</code> | A vector of prior variances for the regression parameters delta (Gaussian priors are assumed) for the zero probability logistic regression component of the model. Defaults to a vector with values 100000. |
| <code>rho</code> | The value in the interval [0, 1] that the spatial dependence parameter rho is fixed at if it should not be estimated. If this argument is NULL then rho is estimated in the model. |
| <code>MALA</code> | Logical, should the function use Metropolis adjusted Langevin algorithm (MALA) updates (TRUE) or simple random walk (FALSE, default) updates for the regression parameters and random effects. Not applicable if <code>family="gaussian"</code> . |
| <code>verbose</code> | Logical, should the function update the user on its progress. |

Value

| | |
|------------------------------|---|
| <code>summary.results</code> | A summary table of the parameters. |
| <code>samples</code> | A list containing the MCMC samples from the model. |
| <code>fitted.values</code> | A vector of fitted values for each area. |
| <code>residuals</code> | A matrix with 2 columns where each column is a type of residual and each row relates to an area. The types are "response" (raw), and "pearson". |

| | |
|---------------------|--|
| modelfit | Model fit criteria including the Deviance Information Criterion (DIC) and its corresponding estimated effective number of parameters (p.d), the Log Marginal Predictive Likelihood (LMPL), the Watanabe-Akaike Information Criterion (WAIC) and its corresponding estimated number of effective parameters (p.w), and the loglikelihood. |
| accept | The acceptance probabilities for the parameters. |
| localised.structure | NULL, for compatability with other models. |
| formula | The formula (as a text string) for the response, covariate and offset parts of the model. If family="zip" this also includes the zero probability logistic regression formula. |
| model | A text string describing the model fit. |
| X | The design matrix of covariates. |

Author(s)

Duncan Lee

References

Leroux B, Lei X, Breslow N (2000). "Estimation of Disease Rates in SmallAreas: A New Mixed Model for Spatial Dependence." In M Halloran, D Berry (eds.), *Statistical Models in Epidemiology, the Environment and Clinical Trials*, pp. 179-191. Springer-Verlag, New York.

Examples

```
#####
#### Run the model on simulated data on a lattice
#####
#### Load other libraries required
library(MASS)

#### Set up a square lattice region
x.easting <- 1:10
x.northing <- 1:10
Grid <- expand.grid(x.easting, x.northing)
K <- nrow(Grid)

#### set up distance and neighbourhood (W, based on sharing a common border) matrices
distance <- as.matrix(dist(Grid))
W <-array(0, c(K,K))
W[distance==1] <-1

#### Generate the covariates and response data
x1 <- rnorm(K)
x2 <- rnorm(K)
theta <- rnorm(K, sd=0.05)
phi <- mvrnorm(n=1, mu=rep(0,K), Sigma=0.4 * exp(-0.1 * distance))
logit <- x1 + x2 + theta + phi
prob <- exp(logit) / (1 + exp(logit))
```

```

trials <- rep(50,K)
Y <- rbinom(n=K, size=trials, prob=prob)

#### Run the Leroux model
formula <- Y ~ x1 + x2
## Not run: model <- S.CARleroux(formula=formula, family="binomial",
trials=trials, W=W, burnin=20000, n.sample=100000)
## End(Not run)

#### Toy example for checking
model <- S.CARleroux(formula=formula, family="binomial",
trials=trials, W=W, burnin=10, n.sample=50)

```

| | |
|----------------|--|
| S.CARlocalised | <i>Fit a spatial generalised linear mixed model to data, where a set of spatially smooth random effects are augmented with a piecewise constant intercept process.</i> |
|----------------|--|

Description

Fit a spatial generalised linear mixed model to areal unit data, where the response variable can be binomial or Poisson. Note, a Gaussian likelihood is not allowed because of a lack of identifiability among the parameters. The linear predictor is modelled by known covariates, a vector of random effects and a piecewise constant intercept process. The random effects are modelled by an intrinsic CAR prior, while the piecewise constant intercept process was proposed by Lee and Sarran (2015), and allow neighbouring areas to have very different values. Further details are given in the vignette accompanying this package. Inference is conducted in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. Missing (NA) values are not allowed in this model. For a full model specification see the vignette accompanying this package.

Usage

```

S.CARlocalised(formula, family, data=NULL, G, trials=NULL, W,
burnin, n.sample, thin=1, prior.mean.beta=NULL, prior.var.beta=NULL,
prior.tau2=NULL, prior.delta=NULL, MALA=FALSE, verbose=TRUE)

```

Arguments

| | |
|---------|---|
| formula | A formula for the covariate part of the model using the syntax of the <code>lm()</code> function. Offsets can be included here using the <code>offset()</code> function. The response, offset and each covariate is a vector of length $K*1$. The response cannot contain missing (NA) values. |
| family | One of either "binomial" or "poisson", which respectively specify a binomial likelihood model with a logistic link function, or a Poisson likelihood model with a log link function. |
| data | An optional data.frame containing the variables in the formula. |

| | |
|-----------------|---|
| G | The maximum number of distinct intercept terms (groups) to allow in the model. |
| trials | A vector the same length as the response containing the total number of trials for each area. Only used if family="binomial". |
| W | A non-negative K by K neighbourhood matrix (where K is the number of spatial units). Typically a binary specification is used, where the jkth element equals one if areas (j, k) are spatially close (e.g. share a common border) and is zero otherwise. The matrix can be non-binary, but each row must contain at least one non-zero entry. |
| burnin | The number of MCMC samples to discard as the burn-in period. |
| n.sample | The number of MCMC samples to generate. |
| thin | The level of thinning to apply to the MCMC samples to reduce their temporal autocorrelation. Defaults to 1 (no thinning). |
| prior.mean.beta | A vector of prior means for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector of zeros. |
| prior.var.beta | A vector of prior variances for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector with values 100000. |
| prior.tau2 | The prior shape and scale in the form of c(shape, scale) for an Inverse-Gamma(shape, scale) prior for tau2. Defaults to c(1, 0.01). |
| prior.delta | The prior maximum for the cluster smoothing parameter delta. Defaults to 10. |
| MALA | Logical, should the function use Metropolis adjusted Langevin algorithm (MALA) updates (TRUE) or simple random walk (FALSE, default) updates for the regression parameters and random effects. |
| verbose | Logical, should the function update the user on its progress. |

Value

| | |
|---------------------|--|
| summary.results | A summary table of the parameters. |
| samples | A list containing the MCMC samples from the model. |
| fitted.values | A vector of fitted values for each area. |
| residuals | A matrix with 2 columns where each column is a type of residual and each row relates to an area. The types are "response" (raw), and "pearson". |
| modelfit | Model fit criteria including the Deviance Information Criterion (DIC) and its corresponding estimated effective number of parameters (p.d), the Log Marginal Predictive Likelihood (LMPL), the Watanabe-Akaike Information Criterion (WAIC) and its corresponding estimated number of effective parameters (p.w), and the loglikelihood. |
| accept | The acceptance probabilities for the parameters. |
| localised.structure | A vector giving the posterior median of which intercept group each area is in. |
| formula | The formula (as a text string) for the response, covariate and offset parts of the model |
| model | A text string describing the model fit. |
| X | The design matrix of covariates. |

Author(s)

Duncan Lee

References

Lee, D and Sarran, C (2015). Controlling for unmeasured confounding and spatial misalignment in long-term air pollution and health studies, *Environmetrics*, 26, 477-487.

Examples

```
#####
#### Run the model on simulated data on a lattice
#####
#### Load other libraries required
library(MASS)

#### Set up a square lattice region
x.easting <- 1:10
x.northing <- 1:10
Grid <- expand.grid(x.easting, x.northing)
K <- nrow(Grid)

#### Split the area into two groups between which there will be a boundary.
groups <- rep(1, K)
groups[Grid$Var1>5] <- 2

#### set up distance and neighbourhood (W, based on sharing a common border) matrices
distance <- as.matrix(dist(Grid))
W <- array(0, c(K,K))
W[distance==1] <-1

#### Generate the response data
phi <- mvrnorm(n=1, mu=groups, Sigma=0.2 * exp(-0.1 * distance))
logit <- phi
prob <- exp(logit) / (1 + exp(logit))
trials <- rep(50,K)
Y <- rbinom(n=K, size=trials, prob=prob)

#### Run the localised smoothing model
formula <- Y ~ 1
## Not run: model <- S.CARlocalised(formula=formula, family="binomial", trials=trials,
G=2, W=W,burnin=20000, n.sample=100000)
## End(Not run)

#### Toy example for checking
model <- S.CARlocalised(formula=formula, family="binomial", trials=trials,
G=2, W=W,burnin=10, n.sample=50)
```

| | |
|-----------------|--|
| S.CARmultilevel | <i>Fit a spatial generalised linear mixed model to multi-level areal unit data, where the spatial random effects have a Leroux conditional autoregressive prior and there are also individual or small group level random effects.</i> |
|-----------------|--|

Description

Fit a spatial generalised linear mixed model to multi-level areal unit data, where the response variable can be binomial, Gaussian or Poisson. The data are structured with individuals within areal units, and different numbers of individuals are allowed within each areal unit. The linear predictor is modelled by known covariates (either individual or areal level) and two vectors of random effects. The latter include areal level effects modelled by the conditional autoregressive prior proposed by Leroux et al. (2000). Independent random effects can be obtained by setting $\rho=0$, while the intrinsic CAR model can be obtained by setting $\rho=1$. The second set of random effects are determined by a single effect for each level of a categorical covariate. These random effects are independent with a common variance σ^2 . Inference is conducted in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. Missing (NA) values are allowed in the response, and posterior predictive distributions are created for the missing values using data augmentation. These are saved in the "samples" argument in the output of the function and are denoted by "Y". For a full model specification see the vignette accompanying this package.

Usage

```
S.CARmultilevel(formula, family, data=NULL, trials=NULL, W, ind.area,
ind.re=NULL, burnin, n.sample, thin=1, prior.mean.beta=NULL, prior.var.beta=NULL,
prior.nu2=NULL, prior.tau2=NULL, prior.sigma2=NULL, rho=NULL, MALA=FALSE,
verbose=TRUE)
```

Arguments

| | |
|---------|---|
| formula | A formula for the covariate part of the model using the syntax of the <code>lm()</code> function. Offsets can be included here using the <code>offset()</code> function. The response, offset and each covariate are vectors with length equal to the number of individuals. The response can contain missing (NA) values. |
| family | One of either "binomial", "gaussian", or "poisson", which respectively specify a binomial likelihood model with a logistic link function, a Gaussian likelihood model with an identity link function, or a Poisson likelihood model with a log link function. |
| data | An optional data.frame containing the variables in the formula. |
| trials | A vector the same length as the response containing the total number of trials for each individual. Only used if family="binomial". |
| W | A non-negative K by K neighbourhood matrix (where K is the number of spatial units). Typically a binary specification is used, where the jkth element equals one if areas (j, k) are spatially close (e.g. share a common border) and is zero otherwise. The matrix can be non-binary, but each row must contain at least one non-zero entry. |

| | |
|------------------------------|--|
| <code>ind.area</code> | A vector of integers the same length as the number of data points (individuals) giving which spatial unit (numbered from 1 to K to align with the rows of the W matrix) each individual belongs to. |
| <code>ind.re</code> | A categorical variable where each level will be assigned a different independent random effect. If NULL these random effects are omitted. |
| <code>burnin</code> | The number of MCMC samples to discard as the burn-in period. |
| <code>n.sample</code> | The number of MCMC samples to generate. |
| <code>thin</code> | The level of thinning to apply to the MCMC samples to reduce their temporal autocorrelation. Defaults to 1 (no thinning). |
| <code>prior.mean.beta</code> | A vector of prior means for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector of zeros. |
| <code>prior.var.beta</code> | A vector of prior variances for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector with values 100000. |
| <code>prior.nu2</code> | The prior shape and scale in the form of $c(\text{shape}, \text{scale})$ for an Inverse-Gamma(shape, scale) prior for nu2. Defaults to $c(1, 0.01)$ and only used if family="Gaussian". |
| <code>prior.tau2</code> | The prior shape and scale in the form of $c(\text{shape}, \text{scale})$ for an Inverse-Gamma(shape, scale) prior for tau2. Defaults to $c(1, 0.01)$. |
| <code>prior.sigma2</code> | The prior shape and scale in the form of $c(\text{shape}, \text{scale})$ for an Inverse-Gamma(shape, scale) prior for sigma2. Defaults to $c(1, 0.01)$. |
| <code>rho</code> | The value in the interval [0, 1] that the spatial dependence parameter rho is fixed at if it should not be estimated. If this argument is NULL then rho is estimated in the model. |
| <code>MALA</code> | Logical, should the function use Metropolis adjusted Langevin algorithm (MALA) updates (TRUE) or simple random walk (FALSE, default) updates for the regression parameters. Not applicable if family="gaussian". |
| <code>verbose</code> | Logical, should the function update the user on its progress. |

Value

| | |
|----------------------------------|--|
| <code>summary.results</code> | A summary table of the parameters. |
| <code>samples</code> | A list containing the MCMC samples from the model. |
| <code>fitted.values</code> | A vector of fitted values for each area. |
| <code>residuals</code> | A matrix with 2 columns where each column is a type of residual and each row relates to an area. The types are "response" (raw), and "pearson". |
| <code>modelfit</code> | Model fit criteria including the Deviance Information Criterion (DIC) and its corresponding estimated effective number of parameters (p.d), the Log Marginal Predictive Likelihood (LMPL), the Watanabe-Akaike Information Criterion (WAIC) and its corresponding estimated number of effective parameters (p.w), and the loglikelihood. |
| <code>localised.structure</code> | NULL, for compatibility with other models. |

| | |
|---------|--|
| formula | The formula (as a text string) for the response, covariate and offset parts of the model |
| model | A text string describing the model fit. |
| X | The design matrix of covariates. |

Author(s)

Duncan Lee

Examples

```
#####
#### Run the model on simulated data on a lattice
#####

#### Set up a square lattice region
x.easting <- 1:10
x.northing <- 1:10
Grid <- expand.grid(x.easting, x.northing)
K <- nrow(Grid)

#### set up distance and neighbourhood (W, based on sharing a common border) matrices
distance <- as.matrix(dist(Grid))
W <- array(0, c(K,K))
W[distance==1] <-1

#### Generate the number of individuals per area and which individuals to which areas
n <- sample(5:30, K, replace=TRUE)
n.total <- sum(n)
ind.area.temp <- rep(1:K, n)
ind.area <- sample(ind.area.temp, n.total, replace=FALSE)

#### Generate a categorical variable
n.levels <- n.total / (0.5*1+0.36*2+0.14*3)
m2 <- round(n.levels * 0.36)
m3 <- round(n.levels * 0.14)
levels.temp <- c(kronecker(1:m3, rep(1,3)), kronecker((m3+1):(m3+m2), rep(1,2)))
m1 <- n.total-length(levels.temp)
levels.temp2 <- c(levels.temp, (max(levels.temp)+1):(max(levels.temp)+m1))
ind.re <-sample(levels.temp2)
q <- m1+m2+m3

#### Generate the covariates and response data
x1 <- rnorm(n.total)
x2 <- rnorm(n.total)
phi <- mvrnorm(n=1, mu=rep(0,K), Sigma=0.4 * exp(-0.1 * distance))
phi.extend <- phi[ind.area]
zeta <- rnorm(n=q, mean=0, sd=0.1)
zeta.extend <- zeta[ind.re]
logit <- x1 + x2 + phi.extend + zeta.extend
prob <- exp(logit) / (1 + exp(logit))
trials <- rep(50,n.total)
```

```

Y <- rbinom(n=n.total, size=trials, prob=prob)

#### Run the model
formula <- Y ~ x1 + x2
## Not run: model <- S.CARmultilevel(formula=formula, family="binomial", ind.area=ind.area,
ind.re=as.factor(ind.re), trials=trials, W=W, burnin=20000, n.sample=100000)
## End(Not run)

#### Toy example for checking
model <- S.CARmultilevel(formula=formula, family="binomial", ind.area=ind.area,
ind.re=as.factor(ind.re), trials=trials, W=W, burnin=10, n.sample=50)

```

S.glm

Fit a generalised linear model to data.

Description

Fit a generalised linear model to data, where the response variable can be binomial, Gaussian, multinomial, Poisson or zero-inflated Poisson (ZIP). Inference is conducted in a Bayesian setting using Markov chain Monte Carlo (MCMC) simulation. Missing (NA) values are allowed in the response, and posterior predictive distributions are created for the missing values via data augmentation. These are saved in the "samples" argument in the output of the function and are denoted by "Y". For the multinomial model the first category in the multinomial data (first column of the response matrix) is taken as the baseline, and the covariates are linearly related to the log of the ratio ($\theta_{j,j} / \theta_{1,1}$) for $j=1, \dots, J$, where $\theta_{j,j}$ is the probability of being in category j . For the ZIP model covariates can be used to estimate the probability of an observation being a structural zero, via a logistic regression equation. For a full model specification see the vignette accompanying this package.

Usage

```

S.glm(formula, formula.omega=NULL, family, data=NULL, trials=NULL, burnin, n.sample,
thin=1, prior.mean.beta=NULL, prior.var.beta=NULL, prior.nu2=NULL,
prior.mean.delta=NULL, prior.var.delta=NULL, MALA=FALSE, verbose=TRUE)

```

Arguments

| | |
|---------|---|
| formula | A formula for the covariate part of the model using the syntax of the <code>lm()</code> function. Offsets can be included here using the <code>offset()</code> function. The response, offset and each covariate are vectors of length $K \times 1$. For the multinomial model the response and the offset (if included) should be matrices of dimension $K \times J$, where K is the number of spatial units and J is the number of different variables (categories in the multinomial model). The covariates should each be a $K \times 1$ vector, and different regression parameters are estimated for each of the J variables. The response can contain missing (NA) values. |
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| <code>formula.omega</code> | A one-sided formula object with no response variable (left side of the "~") needed, specifying the covariates in the logistic regression model for modelling the probability of an observation being a structural zero. Each covariate (or an offset) needs to be a vector of length $K*1$. Only required for zero-inflated Poisson models. |
| <code>family</code> | One of either "binomial", "gaussian", "multinomial", "poisson" or "zip", which respectively specify a binomial likelihood model with a logistic link function, a Gaussian likelihood model with an identity link function, a multinomial likelihood model with a logistic link function, a Poisson likelihood model with a log link function, or a zero-inflated Poisson model with a log link function. |
| <code>data</code> | An optional data.frame containing the variables in the formula. |
| <code>trials</code> | A vector the same length as the response containing the total number of trials for each data point. Only used if <code>family="binomial"</code> or <code>family="multinomial"</code> . |
| <code>burnin</code> | The number of MCMC samples to discard as the burn-in period. |
| <code>n.sample</code> | The number of MCMC samples to generate. |
| <code>thin</code> | The level of thinning to apply to the MCMC samples to reduce their temporal autocorrelation. Defaults to 1 (no thinning). |
| <code>prior.mean.beta</code> | A vector of prior means for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector of zeros. |
| <code>prior.var.beta</code> | A vector of prior variances for the regression parameters beta (Gaussian priors are assumed). Defaults to a vector with values 100000. |
| <code>prior.nu2</code> | The prior shape and scale in the form of $c(shape, scale)$ for an Inverse-Gamma(shape, scale) prior for nu2. Defaults to $c(1, 0.01)$ and only used if <code>family="Gaussian"</code> . |
| <code>prior.mean.delta</code> | A vector of prior means for the regression parameters delta (Gaussian priors are assumed) for the zero probability logistic regression component of the model. Defaults to a vector of zeros. |
| <code>prior.var.delta</code> | A vector of prior variances for the regression parameters delta (Gaussian priors are assumed) for the zero probability logistic regression component of the model. Defaults to a vector with values 100000. |
| <code>MALA</code> | Logical, should the function use Metropolis adjusted Langevin algorithm (MALA) updates (TRUE) or simple random walk (FALSE, default) updates for the regression parameters. Not applicable if <code>family="gaussian"</code> or <code>family="multinomial"</code> . |
| <code>verbose</code> | Logical, should the function update the user on its progress. |

Value

| | |
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| <code>summary.results</code> | A summary table of the parameters. |
| <code>samples</code> | A list containing the MCMC samples from the model. |
| <code>fitted.values</code> | The fitted values based on posterior medians from the model. For the univariate data models this is a vector, while for the multivariate data models this is a matrix. |

| | |
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| residuals | If the family is "binomial", "gaussian" or "poisson", then this is a matrix with 2 columns, where each column is a type of residual and each row relates to an area. The types are "response" (raw), and "pearson". If family is "multinomial", then this is a list with 2 elements, where each element is a matrix of residuals of a different type. Each row of a matrix relates to an area and each column to a category within the multinomial response. The types of residual are "response" (raw), and "pearson". |
| modelfit | Model fit criteria including the Deviance Information Criterion (DIC) and its corresponding estimated effective number of parameters (p.d), the Log Marginal Predictive Likelihood (LMPL), the Watanabe-Akaike Information Criterion (WAIC) and its corresponding estimated number of effective parameters (p.w), and the loglikelihood. |
| localised.structure | NULL, for compatibility with other models. |
| formula | The formula (as a text string) for the response, covariate and offset parts of the model. If family="zip" this also includes the zero probability logistic regression formula. |
| model | A text string describing the model fit. |
| X | The design matrix of covariates. |

Author(s)

Duncan Lee

Examples

```
#####
#### Run the model on simulated data on a lattice
#####
#### Set up a square lattice region
x.easting <- 1:10
x.northing <- 1:10
Grid <- expand.grid(x.easting, x.northing)
K <- nrow(Grid)

#### Generate the covariates and response data
x1 <- rnorm(K)
x2 <- rnorm(K)
logit <- x1 + x2
prob <- exp(logit) / (1 + exp(logit))
trials <- rep(50,K)
Y <- rbinom(n=K, size=trials, prob=prob)

#### Run the model
formula <- Y ~ x1 + x2
## Not run: model <- S.glm(formula=formula, family="binomial", trials=trials,
burnin=20000, n.sample=100000)
## End(Not run)

#### Toy example for checking
```

```
model <- S.glm(formula=formula, family="binomial", trials=trials,
burnin=10, n.sample=50)
```

| | |
|-------------------|---|
| summarise.lincomb | <i>Compute the posterior distribution for a linear combination of the covariates from the linear predictor.</i> |
|-------------------|---|

Description

This function takes in a CARBayes model object and computes the posterior distribution and posterior quantiles of a linear combination of the covariates from the linear predictor. For example, if a quadratic effect of a covariate on the response was specified, then this function allows you to compute the posterior distribution of the quadratic relationship.

Usage

```
summarise.lincomb(model, columns=NULL, quantiles=0.5, distribution=FALSE)
```

Arguments

| | |
|--------------|--|
| model | A CARBayes model object from fitting one of the models in this package. |
| columns | A vector of column numbers stating which columns in the design matrix of covariates the posterior distribution should be computed for. |
| quantiles | The vector of posterior quantiles required. |
| distribution | A logical value stating whether the entire posterior distribution should be returned or just the specified quantiles. |

Value

| | |
|-----------|---|
| quantiles | A 2 dimensional array containing the required posterior quantiles. Each row relates to a data value, and each column to a different requested quantile. |
| posterior | A 2 dimensional array containing the required posterior distribution. Each column relates to a different data value. |

Author(s)

Duncan Lee

Examples

```
## See the vignette accompanying this package for an example of its use.
```

summarise.samples *Summarise a matrix of Markov chain Monte Carlo samples.*

Description

This function takes in a matrix of Markov chain Monte Carlo (MCMC) samples from a CARBayes model object, such as a set of parameters or fitted values, and calculates posterior quantiles and exceedence probabilities. The latter are probabilities of the form $P(\text{quantity} > \text{cldata})$, where c is a threshold chosen by the user.

Usage

```
summarise.samples(samples, columns=NULL, quantiles=0.5, exceedences=NULL)
```

Arguments

| | |
|-------------|---|
| samples | A matrix of MCMC samples obtained from a CARBayes model object. |
| columns | A vector of column numbers stating which columns in the matrix of MCMC samples summaries are required for. Defaults to all columns. |
| quantiles | The vector of posterior quantiles required. |
| exceedences | The vector of threshold levels, c , that exceedence probabilities are required for. |

Value

| | |
|-------------|---|
| quantiles | A 2 dimensional array containing the required posterior quantiles. Each row relates to a parameter and each column to a different requested quantile. |
| exceedences | A 2 dimensional array containing the required exceedence probabilities. Each row relates to a parameter, and each column to a different requested exceedence probability. |

Author(s)

Duncan Lee

Examples

```
## See the vignette accompanying this package for an example of its use.
```


Index

CARBayes (CARBayes-package), [2](#)
CARBayes-package, [2](#)
coef.CARBayes, [4](#)
combine.data.shapefile, [4](#)

fitted.CARBayes, [5](#)

highlight.borders, [6](#)

logLik.CARBayes, [7](#)

model.matrix.CARBayes, [7](#)
MVS.CARleroux, [8](#)

print.CARBayes, [11](#)

residuals.CARBayes, [12](#)

S.CARbym, [13](#)
S.CARDissimilarity, [16](#)
S.CARleroux, [19](#)
S.CARlocalised, [22](#)
S.CARmultilevel, [25](#)
S.glm, [28](#)
summarise.lincomb, [31](#)
summarise.samples, [32](#)